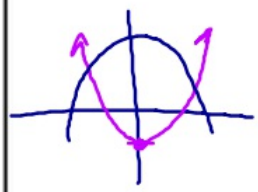
Find the area of the regions enclosed by the lines and curves

24. $x - y^2 = 0$ and $x + 2y^2 = 3$



26. $4x^2 + y = 4$ and $x^4 - y = 1$

Top

$y = 4 - 4x^2$

$-y = 1 - x^4$

$y = x^4 - 1$

$$\int_{-1}^{1} \left[(4 - 4x^2) - (x^4 - 1) \right] dx$$

Intersection

$4 - 4x^2 = x^4 - 1$

$x^4 + 4x^2 - 5 = 0$

$(x^2 + 5)(x^2 - 1) = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

Find the area of the regions enclosed by the lines and curves

18. $y_1 = x^4 - 4x^2 + 4$ and $y_2 = x^2$

$x^4 - 4x^2 + 4 = x^2$

$x^4 - 5x^2 + 4 = 0$

$(x^2 - 4)(x^2 - 1) = 0$

$x = \pm 2 \quad x = \pm 1$

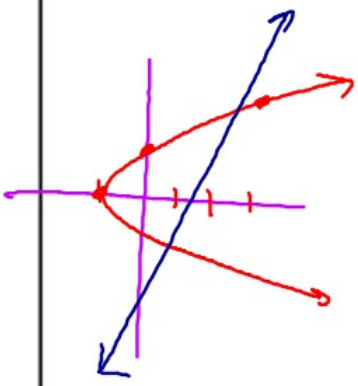
$y(-1.5) =$

$y_1(0) = 4$ Top

$y = 0$ Bottom

$$\int_{-2}^{-1} (y_2 - y_1)\,dx + \int_{-1}^{1} (y_1 - y_2)\,dx + \int_{1}^{2} (y_2 - y_1)\,dx$$

23. $y^2 - 4x = 4$ and $4x - y = 16$



$y = -16 + 4x$

$y = \pm\sqrt{4 + 4x}$

$\frac{4x}{4} = \frac{16}{4} + \frac{y}{4}$

$x = 4 + \frac{1}{4}y$

$\frac{-4x}{-4} = \frac{4}{-4} - \frac{y^2}{-4}$

$x = -1 + \frac{1}{4}y^2$

$4\left(-1 + \frac{1}{4}y^2 = 4 + \frac{1}{4}y\right)$

$-4 + y^2 = 16 + y$

$y^2 - y - 20 = 0$

$(y - 5)(y + 4)$

$$\int_{-4}^{5} \left(4 + \frac{1}{4}y\right) - \left(-1 + \frac{1}{4}y^2\right)$$

What you'll Learn About

- Finding lengths of curves

2. Use your calculator to find the length of the curve

$y = \tan x \quad \frac{-\pi}{3} \le x \le 0$

$L = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}$

$\frac{dy}{dx} = \sec^2 x$

$L = \int_{-\frac{\pi}{3}}^{0} \sqrt{1 + (\sec^2 x)^2}\, dx$

$L = 2.056$

4. Use your calculator to find the length of the curve

$x = \sqrt{1-y^2} \quad \frac{-1}{2} \le y \le \frac{1}{2}$

$L = \int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$

$x = (1-y^2)^{1/2}$

$\frac{dx}{dy} = \frac{1}{2}(1-y^2)^{-1/2} \cdot -2y$

$= \frac{-y}{\sqrt{1-y^2}}$

$\left(\frac{dx}{dy}\right)^2 = \frac{y^2}{1-y^2}$

$L = 1.047$

8. Use your calculator to find the length of the curve

$x = \int_0^{y^2} \sqrt{\sec^2 t - 1} \quad \frac{-\pi}{3} \le y \le \frac{\pi}{4}$

$L = \int_{-\frac{\pi}{3}}^{\pi/4} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}$

$\frac{dx}{dy} = 2y\sqrt{\sec^2(y^2) - 1}$

$\left(\frac{dx}{dy}\right)^2 = 4y^2(\sec^2 y^2 - 1)$